On the Reality of the Accelerating Universe

Attila Mészáros

Astronomical Institute of the Charles University, V Holešovičkách 2, 180 00 Prague 8, Czech Republic

ABSTRACT

Two groups recently deduced the positive value for the cosmological constant, concluding at a high ($\geq$ 99%) confidence level that the Universe should be accelerating. This conclusion followed from the statistical analysis of dozens of high-redshift supernovae. In this paper this conclusion is discussed. From the conservative frequentist's point of view the validity of null hypothesis of the zero cosmological constant is tested by the classical statistical χ^2 test for the 60 supernovae listed in Perlmutter et al. 1999 (ApJ, 517, 565). This sample contains 42 objects discovered in the frame of Supernova Cosmology Project and 18 low-redshift object detected earlier. Excluding the event SN1997O, which is doubtlessly an outlier, one obtains the result: The probability for seeing a worse χ^2 - if the null hypothesis is true - is in the 5% to 8% range, a value that does not indicate significant evidence againts the null. If one excludes further five possible outliers, proposed to be done by Perlmutter et al. 1999, then the sample of 54 supernovae is in an excellent accordance with the null hypothesis. It also seems that supernovae from the High-z Supernova Search Team does not change the acceptance of null hypothesis. This means that the rejection of the Einstein equations with zero cosmological constant - based on the supernova data alone - is still premature.

Subject headings: supernovae: general – cosmology: miscellaneous

1. Introduction

In recent years two independent groups (Perlmutter et al. (1999), Riess et al. (2000) and references therein) concluded that the cosmological constant is positive with $\Omega_\Lambda \simeq 0.7$ and $\Omega_M \simeq 0.3$ (for a detailed review and further references see, e.g., Riess (2000); for the latest developments see Riess et al. (2001)). As usual, Ω_M denotes the ratio of the density of the non-relativistic matter in Universe to the critical density; $\Omega_\Lambda = \lambda c^2/(3H_o^2)$, where

λ is the cosmological constant, c is the velocity of light, and H_o is the Hubble constant. This conclusion was based purely on the data of the observations done in the Supernova Cosmology Project (Perlmutter et al. (1999) and references therein) and of the observations done by the High-z Supernova Search Team (Schmidt et al. (1998), Riess et al. (2000) and references therein). The Universe should also be accelerating, because $\Omega_\Lambda > \Omega_M/2$ (see Riess (2000) for more details).

Both teams recognize that the supernovae at redshift $z \simeq (0.3 - 1.0)$ give in average a $\simeq 0.28$ mag bigger distance moduli than expected if $\Omega_M \simeq 0.3$ and $\Omega_\Lambda = 0$ (Riess 2000). This excess of distance modulus is so small and there are so many sources of uncertainties that extreme care is needed in drawing conclusions. This fact is, of course, clearly proclaimed by both teams. Therefore, further careful analysis concerning the methods, statistics, errors, alternative explanations, etc... are highly required. Any new result - even of minimal technical importance - is highly desirable and should immediately be announced (R. Kirshner, private communication). For example, Drell, Loredo & Wasserman (2000) and Gott et al. (2001) gave smaller evidence for the non-zero cosmological constant.

This article is - in essence - also such a required contribution. It discusses one concrete question of the topic; namely, the probability of the rejection of the zero cosmological constant hypothesis. The discussion is done from the pure statistical point of view.

2. General considerations

In Perlmutter et al. (1997) and Perlmutter et al. (1999) (in what follows P99) their analysis of the data gives the conclusion that $\Omega_\Lambda > 0$ holds with a 99% confidence. In Riess et al. (2000) a higher than 99% confidence is deduced. Gott et al. (2001) deduced - from an earlier sample - that the confidence for $\Omega_\Lambda > 0$ is only 89.5% or smaller. All these statistical analyses followed the so called "Bayesian approach". The key idea of this approach is based on the procedure, in which - even before existing any measured data concerning a hypothesis - some preliminary degree of plausability ("Bayesian prior") is assigned to the hypothesis (for more details about the Bayesian approach in Astronomy see, cf., Drell, Loredo & Wasserman (2000) and references therein; about the different aspects of methods from the statistical point of view see Berger (2002)). In the case of the supernovae the different confidence levels came from the different prior of the hypothesis $\Omega_\Lambda = 0$.

The author means that it is highly useful to provide an analysis of data from the frequentist's point of view, too. This approach proceeds classically and most conservatively. This means that - at the begining - it is simply assumed that the Friedmannian model (either with $\Omega_M > 1$ or with with $\Omega_M = 1$ or with $0 < \Omega_M < 1$) *with zero cosmological constant* is the correct model. Then it is asked that the observational data are in accordance with this model or not (for more details concerning this statistical approach see any standard text-book of Statistics (Trumpler & Weaver 1953; Kendall & Stuart 1976); from newer publications see, e.g., Feldman & Cousins (1998) and references therein).

The requirement of this analysis may be supported as follows.

It is a standard knowledge that the Friedmannian model with $\Omega_\Lambda = 0$ is based on two different assumptions: A. Gravitation is described by the Einstein equations with zero cosmological constant. B. The Universe has a symmetry defined by six linearly independent Killing vectors, and the character of this symmetry allows to speak about a maximally symmetric three-dimensional sub-manifold (this assumption is called as "cosmological principle"; for more details see, cf. Weinberg (1972); Chapt. 14.1).

The verification of these assumptions should be done by observations, of course. In the verification of the assumption B. it is quite usual to proceed in the frame of the most conservative point of view. In addition, theoretically, even if the cosmological principle were rejected, it would not be clear, which non-Friedmannian model should then be used (for the survey of non-Friedmannian models see, cf., Kraśiński (1997)). Simply, if the cosmological principle is not rejected yet unambiguously at a high significance, then the best is "to keep the cosmological principle as far as possible".

The author means that one should similarly proceed concerning the assumption A., too. From the observational point of view Drell, Loredo & Wasserman (2000) and Gott et al. (2001) suggest that one should remain careful in the final conclusions. In addition, from the theoretical point of view, even if the observations were rejecting assumption A., one would be able to introduce *several different generalizations* of Einstein equations. For example, Gott et al. (2001) discusses both the usual generalization with cosmological constant, but also the possibility with the time-variable "constant". This second possibility is identical to the introduction of a long-range scalar field coupled with the gravitation. In fact, there are known many similar theoretical attempts for other fields coupled with the gravitation (see Gott et al. (2001) and references therein). Add here that also the author probed to introduce such long-range force defined by a pair of standard spin-2 fields (Mészáros 1987). This probe was proclaimed to be hopeless, because of the unsolvable complications in the theory (Mészáros 1991). In any case, the introcution of non-zero cosmological constant is not the only possible generalization of Einstein equations. Simply, also here the best choice is "to keep the assumption A. as far as possible".

The observations quite unambiguously suggest that $\Omega_M \geq 1$ is excluded and, in addition, it may be assumed $0.1 \leq \Omega_M \leq 0.5$ (Bahcall & Fan 1998). Hence, the null hypothesis will be the assumption of the correctness of the Friedmannian model with $0.1 \leq \Omega_M \leq 0.5$ and $\Omega_\Lambda = 0$. The sample obtained from observations will be given by the 60 supernovae collected and discussed at P99. Hence, the precise purpose of this article is to test: Does this sample alone reject the null hypothesis? This is studied in Section 3. The remaining supernovae from the second project, and also some other questions, will shortly be discussed in Section 4. In Section 5 the results of paper will be summarized.

3. The χ^2 test

Be given the data of 60 supernovae collected at Table 1 and Table 2 of P99. Then one has to fit the $[m_B^{eff}, \log z]$ data-pairs with the theoretical curves, in which $\Omega_\Lambda = 0$ holds *identically.* This means that there are only two independent parameters in these theoretical curves (H_o and Ω_M). The procedure is a standard one, and is described, e.g., by Press et al. (1992), Chapt.15.1. One has to do three things: To determine the two best-fit parameters; to determine their allowed ranges; to determine the goodness-of-fit due to the standard χ^2 test. Eq. 15.1.5 of Press et al. (1992) takes the form

$$\chi^2 = \sum_{i=1}^{N} \Big(\frac{m_{Bi}^{eff} - m_B^{eff}(z_i, H_o, \Omega_M, \Omega_\Lambda = 0)}{\sigma_i} \Big)^2, \tag{1}$$

where N is the number of supernovae in the sample, and σ_i is the uncertainty of the effective magnitude of i-th supernova having measured the redshift z_i and corrected B band magnitude m_{Bi}^{eff} $(i = 1, 2, ..., N)$. The corrected B band magnitude is given by

$$m_B^{eff} = 25 + M_B + 5\log(c/H_o) + 5\log Q(z, \Omega_M, \Omega_\Lambda = 0), \tag{2}$$

where M_B is the absolute magnitude, c/H_o is in Mpc, and one has (Carroll, Press & Turner 1992)

$$Q(z) = (2/\Omega_M^2)(2 + \Omega_M(z-1) - (2-\Omega_M)\sqrt{1+\Omega_M z}). \tag{3}$$

This standard relation of Cosmology is obtainable also directly (Mészáros & Mészáros 1996) without the integration of general equation presented by Carroll, Press & Turner (1992). The null hypothesis should then be rejected in the case when either the best-fit parameters are fully unphysical or the goodness-of-fit excludes the fit itself. In our case we will proceed in such a way that only the observationally allowed ranges of parameters will be considered - hence, if one obtains a good fit from the goodness-of-fit, then the fit is immediately acceptable.

In our case $N = 60$, and one may take in accordance with Perlmutter et al. (1997) $\tilde{M} = M - 5\log H_o + 25 = -3.32$. A $\simeq 12\%$ observational uncertainty in the value of H_o (Gott et al. 2001; Freedman et al. 2001) gives maximally a $\simeq 5\log 1.12 = 0.28$ mag change in the value of $\tilde{M}$. This means that in the range $3.60 > -\tilde{M} > 3.04$ one should search for the best fits.

Using the measured redshifts, corrected effective magnitudes and their uncertainties one obtains the best fit for $\Omega_M = 0.1$, and for $\tilde{M} = 3.30$; namely $\chi^2 = 107.1$. This value is the best fit for 58 degrees of freedom.

Varying the free parameters in the allowed ranges one obtains the following. If $\tilde{M} = -3.32$ and $\Omega_M = 0.1$, one has $\chi^2 = 108.0$. In fact, in all fits of this Section the best fits for $\tilde{M}$ were practically always given by $\tilde{M} = -3.32$. The values of χ^2 obtained for $\tilde{M} = -3.32$ and for the best fits values of $\tilde{M}$ gave practically the same significance levels - the differences were smaller than 1%, which is unimportant for the purpose of this paper. Therefore, in what follows, the value of $\tilde{M} = -3.32$ may always be taken as the best fit value. In the case of parameter Ω_M the worst fit is obtained for $\Omega_M = 0.5$, namely $\chi^2 = 129.6$. Between $\Omega_M = 0.5$ and $\Omega_M = 0.1$ the fitting is monotonously strengthening, if one goes toward the smaller values. Contrary to $\tilde{M}$, in the case of parameter Ω_M, the best fit value is on the boundary of the allowed range of parameter.

The goodness-of-fit is given by the chi-square probability function $P(\nu/2, \chi^2/2)$ (cf. Press et al. (1992), Chapt.6.2) for $\nu = 58$ degrees of freedom. Add here that fast approximate probability of the goodness-of-fit is obtainable also without the calculation of this function directly from the table of χ^2 distribution (see Trumpler & Weaver (1953), Table A5). One may use the fact that roughly for $\nu > 20$ degrees of freedom the reduced χ^2/ν distribution is practically not changing. For $\nu = 58$ and $\chi^2 = 108$ the significance level is between 1% and 0.1%. For $\Omega_M = 0.5$ the fit even is worse, and the significance level is around 0.1%.

For the sake of statistical precisity two notes must be added here.

The first one concerns the degrees of freedom. In fact, m_B^{eff} itself is a corrected value in P99, and contains two further parameters (see P99 for more details). Hence, the degree of freedom, as it seems, for 60 objects should be $\nu = 56$. In fit A of Table 3 in P99 this values is used. Further complications can come from the fact that the best fit value $\Omega_M = 0.1$ is a boundary value of allowed range. This may cause some problems (for a discussion of this question see, for example, Protassov et al. (2002)). In our case this may mean that - in essence - Ω_M should not be considered as a free parameter, but the value $\Omega_M = 0.1$ should be fixed immediately. In this case the degree of freedom should be increased by one. All this means that there is an ambiguity in the concrete value of the degree of freedom. Furtonately, in our case, this problem is not essential: The significances are the same - the difference is smaller than 1%, once the degree of freedom is changed by one. Therefore, in what follows, we may further take $\nu = N - 2$ for N objects.

The second notes concerns the errors. Strictly speaking, one should include into σ_i also the errors of $\log z$ (see Press et al. (1992), Chapt. 15.3). But the errors in $\log z$ - compared with the errors of magnitudes - are small (except for some low redshift objects). In any case, these additional errors should decrease the significances of rejection, because they should decrease the value of χ^2. But this effect should also be unimportant here.

The approximate significance from the reduced χ^2/ν, the effect of errors in redshifts, the boundary value of Ω_M together with the ambiguity in the degree of freedom, and the choice $\tilde{M} = -3.32$ may cause a maximally $(1-3)\%$ uncertainty in the obtained significance. This inpreciseness is inessential for our purpose.

For our purpose it is essential that the sample with $N = 60$ supernovae gives a fully wrong fit. *The null hypothesis for the whole sample should be rejected; the significance level is in the range* $(0.1 - 3.0)\%$*, being enough to reject the null.*

Nevertheless, doing a final conclusion, a care is still needed. This follows from the following fact. An inspection of terms in $\chi^2 = 108$ for $\Omega_M = 0.1$ shows that in this sum a big amount, namely 26.7, is given by one supernova, namely by SN1997O having $z = 0.374$. Hence, if this one single object were not considered in the sample, then the sample with $N = 59$ object would give only $\chi^2 = 81.3$ for $\nu = 57$ degrees of freedom. This would already be an acceptable fit, because the rejection of null hypothesis would occuring at a $(6-7)\%$ significance level. Taking into account also the possible $(1-3)\%$ uncertainty, one may conclude that the the usually requested $< 5\%$ significance level - allowing the rejection - should not be reached.

Hence, we arrive at the surprising result: *The null hypothesis is rejected; but by one single object!*

There are three different arguments suggesting that SN1997O should actually be removed from the sample.

The first argument comes from general statistical considerations of outliers. It is never strange in Statistics to remove an object from the sample, if this is an "outlier". Generally, outliers are observations which are inconsistent with the remainder of the data set (a detailed discussion of outliers is given, e.g., by Jolliffe (1986); Chapt.10.1). Looking into Figures 1 and 2 of P99, one immediately sees that just SN1997O is a good candidate for an outlier, because it is far above the magnitudes expected from the trend given by other objects. The object is "too faint".

The second argument follows from the text of P99. This article also discusses the question of outliers from astrophysical point of view (different light curves, reddening in the host galaxy, etc.). Four supernovae, namely SN 1992bo, 1992bp, SN1994H, SN1997O, are proclaimed as "most significant outliers". There are further two ones (SN1996cg, SN1996cn), which are also proposed not to be taken into the sample from different reasons. Hence, also P99 takes SN1997O as an outlier, too. In addition, further three or five objects are proposed to be removed, too.

The third argument follows the following consideration. Assume that no outliers are in the sample. Then the null hypothesis is rejected, and the generalization of Einstein equations is needed. There are several possibilities for this generalization. One of this is the non-zero cosmological constant. Then one should fit the whole sample with $N = 60$ with the theoretical curves allowing $\Omega_\Lambda \neq 0$. This was already done by P99 (Table 3, fit A); the value $\chi^2 = 98$ was obtained. But this is *again a wrong fit*, because for 58 degrees of freedom one obtains a *rejection* at the significant level 1% (Trumpler & Weaver 1953). All this means that in this case *both* $\Omega_\Lambda = 0$ *and* $\Omega_\Lambda \neq 0$ *should be rejected.* Simply, also the generalization with non-zero cosmological constant is *not* acceptable. It is even questionable that any theoretical curve - in the frame of cosmological principle - can fit this sample (see Weinberg (1972), Chapt. 14 for the general discussion of theoretical curves). Hence, the object SN1997O *alone* should reject the Einstein equations both with zero and non-zero cosmological constant; in addition, probably also the cosmological principle itself.

The author means - in accordance with P99 - that this object is a clear outlier and should be removed from the sample. All this means that the best is to consider three different samples. The first one is the sample with $N = 59$ objects removing only SN1997O. The second sample is the sample B of P99; the third one is the "primary sample" of P99 having $N = 54$ objects (sample C). P99 proposes to use this third sample as the best primary choice.

The first sample with $N = 59$ gives an acceptable fit; the rejection of null hypothesis should be at the significance level $(5-8)$%. The second sample with $N = 56$ gives $\chi^2 = 68.3$ for $\Omega_M = 0.1$, which is again an acceptable fit for $\nu = 54$ degrees of freedom. The null hypothesis is rejected at 11% significance level. The primary sample with $N = 54$ gives $\chi^2 = 63.7$ for $\Omega_M = 0.1$. This value for $\nu = 52$ degrees od freedom gives an excellent fit - the null hypothesis is rejected at the 28% significance level. In any case, $< 5\%$ level is never reached.

4. Discussion

In Riess et al. (1998) (see also Riess et al. (1999)) there are discussed 10 further high-redshift supernovae with $0.16 \leq z \leq 0.62$. Of course, the best solution would be to fit these objects together with the 60 objects of P99. Nevertheless, the errors in Riess et al. (1998) are listed in other way than in P99. In addition - even having a list of σ_i for any object obtained by the same manner - further complications can arise from the existence of outliers. (Clearly, the same criterion for an outlier should be required in such a "matched" sample. It is not clear, how to define this criterion.) Simply, the matching of the all possible observed supernovae into one single statistical sample leads to several technical problems, and the author - not being in the teams of two projects - is not able to solve this technical question. Therefore, here, the supernovae from the second team will be fitted separately.

Using Eq.4 of Riess et al. (1998), in which $\Omega_\Lambda = 0$ and $\sigma_v = 0$, one may provide the fitting for 10 objects listed in Table 5 and Table 6 of Riess et al. (1998). Taking the values of μ_o and σ_{μ_o} from the last column of Table 5, and taking the possible values of free parameter H_o (in units km s^{-1} Mpc^{-1}) between 64 and 80 (Freedman et al. 2001), one obtains the best fit for $H_o = 79$ km s^{-1} Mpc^{-1}, and $\Omega_M = 0.1$; namely $\chi^2 = 9.03$. Taking the values of μ_o and σ_{μ_o} from the last column of Table 6, one obtains the best fit for $H_o = 78$ km s^{-1} Mpc^{-1}, and $\Omega_M = 0.1$; namely $\chi^2 = 7.7$. Both cases are excellent fits, because the significance level is around 40% and 50%, respectively, for 8 degrees of freedom (Trumpler & Weaver 1953). Note here that the change caused by Ω_M is weak. For example, in the first case for $\Omega_M = 0.5$ one still has $\chi^2 = 9.6$. The dependence on the change of H_o is more essential, but in any case for $H_o = 77 - 80$ km s^{-1} Mpc^{-1} acceptable fits are obtained. The choice $\sigma_v = 0$ is not a problem, because eventual non-zero values further decrease the value of χ^2 and thus further strengthen the goodness of fits. The 10 supernovae from Riess et al. (1998) *alone* do not need non-zero cosmological constant.

For the sake of completeness the object SN1997ff with $1.5 < z < 1.8$ should also be discussed (Riess et al. 2001). For this object the uncertainty at $\Delta(m - M)$ is so large ($\simeq 1$ mag, as this is clear from Figures 10 and 11 of Riess et al. (2001)) that here the contribution for χ^2 should surely be smaller than 1. This object alone should even strengthen the acceptance of null hypothesis.

Discussing the results of article it must still be precised the following. Strictly speaking, this article does not claim that the introduction of non-zero cosmological constant cannot be done. It is only said that - purely from the most conservative statistical point of view and purely from the supernovae observational data *alone* - the assumption of zero cosmological is *not* rejected yet at a high enough significance level. The reality of the non-zero cosmological constant is not excluded; it remains an open problem yet. In any case, the different statistical methods - either from the Bayesian (Drell, Loredo & Wasserman 2000; Gott et al. 2001) or from frequentist's point of view (this article) - still suggest that - from the statistical point of view - the "definite", "final" or "unambiguous" introduction of non-zero cosmological term - based on the supernova data alone - is still premature. In fact, this is the key result of this article.

5. Conclusions

The results of paper may be summarized as follows.

1. The observational data of 60 supernovae - listed in P99 - were reanalyzed from the conservative statistical point of view. The null hypothesis of zero cosmological constant is not rejected by these data alone. The probability for seeing a worse χ^2 - if the null hypothesis is true - is in the 5% to 28% range, a value that does not indicate significant evidence againts the null. If only one clear outlier is omitted, then this probability is $(5-8)$%; if further outliers - proposed by P99 - are omitted, then this probability is $(10-28)$%. The value < 5% is not reached.

2. The High-z Supernova Search Team data alone suggest that this conclusion further holds.

3. All this means that the introduction of non-zero cosmological constant - based on the supernovae data alone - is still premature. The reality of the non-zero cosmological constant remains an open question.

The author thanks the valuable discussions with L.G. Balázs, A. Graham, R. Kirshner, J. Lub and M. Wolf. The useful remarks of an anonymous referee are kindly acknowledged. This research was supported by Czech Research Grant J13/98: 113200004.

REFERENCES

Bahcall, N.A., & Fan, X. 1998, ApJ, 504, 1

Berger, J. 2002, http://www.isds.duke.edu/ berger/papers/02-01.html

Carroll, S.M., Press, W.H. & Turner, E.L. 1992, ARA&A, 30, 499

Drell, P.S., Loredo, T.J. & Wasserman, I. 2000, ApJ, 530, 593

Feldman, G.J. & Cousins, R.D. 1998, Phys.Rev.D, 57, 3873

Freedman, W.L. et al 2001, ApJ, 553, 47

Gott, J.R., Vogeley, M.S., Podariu, S., & Ratra, B. 2001, ApJ, 549, 1

Jolliffe, I.T. 1986, Principal Component Analysis (Springer, New York)

Kendall, M., & Stuart, A. 1976, The Advanced Theory of Statistics (Griffin, London)

Kraśiński, A. 1997, Inhomogeneous cosmological models (Cambridge Univ. Press, Cambridge)

Mészáros, A. 1987, Phys.Rev., D35, 1176

Mészáros, A. 1991, Gen.Rel.Grav., 22, 417

Mészáros, A. & Mészáros, P. 1996, ApJ, 466, 29

Perlmutter, S. et al. 1997, ApJ, 483, 565

Perlmutter, S. et al. 1999, ApJ, 517, 565 (P99)

Press, W.H., Teukolsky, S.A., Vetterling, W.T., & Flannery, B.P. 1992, Numerical Recipes in FORTRAN (Cambridge Univ. Press, Cambridge)

Protassov, R., van Dyk, D.A., Connors, A., Kashyap, V.L., & Siemiginowska, A. 2002, ApJ, to appear (astro-ph/0201547)

Riess, A.G. 2000, PASP, 112, 1284

Riess, A.G. et al. 1998, AJ, 116, 1009

Riess, A.G. et al. 1999, AJ, 117, 707

Riess, A.G. et al. 2000, ApJ, 536, 62

Riess, A.G. et al. 2001, ApJ, 560, 49

Schmidt, B.P. et al. 1998, ApJ, 507, 46

Trumpler, R.J., & Weaver, H.F. 1953, Statistical Astronomy (University of California Press, Berkeley)

Weinberg, S. 1972, Gravitation and Cosmology (J. Wiley & Sons., New York)